Liquidity risk and contagion for liquid funds

Serge Darolles[*] Jérémy Dudek[†] Gaëlle Le Fol[‡]

First draft: November 22, 2011

This version: October 25, 2013

Abstract

Fund managers face liquidity problems but they have to distinguish the market liquidity risk implied by their assets and the funding liquidity risk. This latter is due to both the liquidity mismatch between assets and liabilities and the redemption risk due to the possible outflows from clients. The main contribution of this paper is the analysis of contagion looking at common market liquidity problems to detect funding liquidity problems. Using the CDS Bond Spread basis as a liquidity indicator and a state space model with time-varying volatility specification, we show that during the 2007-2008 financial crisis, there exist pure contagion effects both in terms of price and liquidity on the emerging sovereign debt market. This result has strong implication since the main risk for an asset manager is to get stuck with an unwanted position due to a dry-up of market liquidity.

JEL classification: G01, G15, C01, C32

Key words: Emerging Markets, Sovereign Debt Market, Liquidity Risk Management, Liquidity, Contagion Effects, Regime Switching models.

[*]Université Paris Dauphine - DRM, France, serge.darolles@dauphine.fr

[†]CREST-INSEE and Université Paris Dauphine - DRM, France, jeremy.dudek@ensae.fr

[‡]Université Paris Dauphine - DRM and CREST-INSEE, France, gaelle.le_fol@dauphine.fr.

The authors gratefully acknowledge financial support of the chair Amundi/Dauphine Fundation "Asset Management". We thank Viral Acharya, Monica Billio, Patrick Gagliardini, Christian Gourieroux, Dale Rosenthal, Sessi Tokpavi, the participants of the 5^{th} International conference on Computational and Financial Econometrics in London, the 4^{th} Annual Conference on Hedge Funds in Paris, the 66^{th} European Meeting of the Econometric Society in Malaga, the 6^{th} Financial Risks International Forum in Paris, the finance department seminar, New York University-STERN School of Business, the finance seminar, CREST for useful comments. Dudek gratefully acknowledges Viral Acharya for sponsoring NYU visiting position in 2012.

1 Introduction

The main difference between traders and fund managers come from their sources of funding. Indeed, traders are funded by banks while fund managers are funded directly by investors. Moreover, we are able to distinguish the liability part of the fund which consists of the investor's inflows and the asset part which contains the fund's holdings. Consequently, the fund managers face a liquidity mismatch between the asset and the liability sides. On the one hand, investors want more and more liquid exposures. As a result, the liquidity of the liabilities is contractually defined and usually very high. On the other hand, for the asset side of the fund, the liquidity is determined by the nature of investments and usually lower than that of the liabilities. As the behavior of these funding providers can largely differ, the fund managers need to monitor this liquidity mismatch. Increasing the cash balance of the fund is one way to minimize this problem. However, if the amount of cash is too large, it will be idle and not producing. Conversely, if it is too small, the fund will still be exposed to the liquidity risk so that, it would be useless. In addition to the mismatch of liquidity that the fund managers suffer, we know that investors need funding to trade securities. When the funding liquidity conditions are bad, they cannot easily access to capitals which impair their trade capacities. If many investors are concerned by such a funding liquidity problem, trading is slowing down and market liquidity reduces[1]. As investors' funding also depends on assets' market liquidity, these problems can be mutually reinforced leading to liquidity spirals [see e.g., Gromb and Vayanos (2002), Morris and Shin (2004), Brunnermeier and Pedersen (2009), Menkveld and Wang (2011)]. Indeed, during financial turmoil, like in 2008, crises can spread across assets and markets as many investors were seeking for liquidity creating a contagion effects.

In this paper, we compute a market liquidity indicator and we define a funding liquidity problem based on a simultaneous constraint of market liquidity for every markets. In order to control for the liquidity risk, we have to define how to measure it but despite the large number of liquidity measures available[2], measuring liquidity remains a difficult task. In fact, many

[1] Brunnermeier and Pedersen (2009) distinguish funding liquidity from market liquidity. The former characterizes the possibility for traders to find funds while the second characterizes the ease to trade an asset on the market. Traders provide market liquidity and their ability to do so depends on their capacity of funding. Funding liquidity is binding market liquidity as traders can only provide liquidity if they can access to fundings.

[2] Aitken and Winn (1997) report more than 68 measures for market liquidity.

liquidity measures require the use of high-frequency transactions and quotes data, which may not be available for some markets and even more so for emerging markets. Goyenko et al. (2009) compare the performances of several liquidity measures relatively to the effective or realized bid-ask spread. However, the poor availability of data encourages us to focus only on liquidity measures based on price data. There exist few measures based on daily price data. Roll (1984) proposes an estimation of the effective bid-ask spread based on the serial covariance of daily price changes. Hasbrouck (2004) uses a Bayesian estimation approach to estimate the Roll model and proposes a Gibbs measure of liquidity. Lesmond et al. (1999) use the proportion of zero return days as a proxy for liquidity. In the line of Levy (2009), we use the Credit Default Swap (CDS hereafter) Bond Spread basis as a liquidity indicator. We use an arbitrage relation to extract a liquidity measure of the sovereign debt market that solely relies on price data. From Garleanu and Pedersen (2009), Fontana and Scheicher (2010) and Bai and Collin-Dufresne (2011), we know that the basis is related to the credit risk of a bond. In other words, a larger deviation from parity is found for lower rated bonds because it is more costly to finance the arbitrage trade. In this paper, we tackle the problem of different currencies into the CDS Bond spread basis measurement and we focus on its liquidity component. Then, we use a Regime Switching Dynamic Correlation model (or RSDC) in order to define whether contagion effects occur. In this model, both heteroscedasticity problem and exogenous definition of crisis dates are tackled using a GARCH model and a regime switching governed by a Markov chain, respectively.

The contribution of this paper is to propose a methodology able to extract a funding liquidity factor from the market liquidity indicators. We establish the occurrence of pure contagion effects on the CDS Bond Spread basis and define them as liquidity contagion. Indeed, as Adrian and Shin (2008) described, contagion is not anymore only modeled as a domino's fall. Effectively, the contagion refers to the transmission of shocks. In this case, the channel of transmission is usually the price of the assets and the financial contagion is often represented as multiple sequential bankruptcies of financial institutions. But the financial crisis of 2007-08 exhibits the fact that financial shocks are also transmitted through liquidity problems. This channel represents a new kind of contagion that we define as the liquidity contagion and that us represented by an increase of correlations between market liquidity risks. Then, we propose to associate these phenomena of liquidity contagion with the identification of a funding liquidity problem. Indeed, although

Brunnermeier and Pedersen (2009) determine a link between market and funding liquidity, it remains difficult to model it. First of all, in order to study funding liquidity, we need to isolate one of its three components: (i) margin risk, (ii) roll over risk and (iii) redemption risk. Considering the case of index funds, we only focus on the last one; the redemption risk implied by the behavior of fund clients. We focus on index funds tracking the performance of emerging sovereign debt markets. They do not use neither leverage nor derivatives leading to only have redemption risk by clients who want a very liquid exposure to particularly illiquid sovereign debt.

Until 2008, the financial sector almost only controls for market risk. The fund managers being constrained to build a portfolio to benefit from the diversification principle have as main fear the re-correlation of their assets. However, in addition to this risk, fund managers face particular liquidity constraints defined in the characteristics of the fund. These funding liquidity problems alter the ability of the manager to finance their trades. Indeed, the flow of clients is a key driver of the funding liquidity of a fund manager. Nonetheless the funding liquidity is also very worrying for both clients and regulators. The first wants to know the exposition of the fund and determine whether this risk is priced into the returns of the fund. The second has to improve the regulation taking into account funding liquidity and consequently, the regulator looks for a relevant indicator. Finally, the funding liquidity is at the very center of the preoccupations of many people and has strong implications both in terms of asset management and regulation.

The remainder of the paper is organized as follows. Section 2 presents the data and the motivation of focusing on the Emerging Markets. Section 3 introduces the CDS Bond Spread Basis, the market liquidity measure and the methodology for applying it in the case of multiple currencies. Section 4 describes the results about the liquidity contagion and its implications. Finally, section 5 proposes robustness check and additional results before the conclusion of the paper in section 6.

2 Data

In this section, we present the interest of focusing on the Emerging Markets and especially the sovereign debts when we study funding liquidity problems. Then, we introduce the database needed in order to compute our liquidity indicator and the sample of countries for which we explore the possible presence of contagion effects.

2.1 Emerging Sovereign Debt Markets

The term of *Emerging Markets* (EM hereafter) appears for the first time in 1981. Since then the World Bank classifies as EM any markets meeting at least one of the following criteria: (i) being located in a low or middle-income economy as defined by the World Bank, (ii) not exhibiting financial depth; the ratio of the country's market capitalization to its Gross Domestic Product (GDP) is low[3], (iii) existence of broad based discriminatory controls for non-domiciled investors, or (iv) being characterized by a lack of transparency, depth, market regulation, and operational efficiency. The creation of emerging markets is motivated by the need of developing countries to raise capital to finance their growth. Before the 2000's developing countries borrowed either from commercial banks or from foreign governments multilateral lenders (International Monetary Fund or World Bank). Capital flows to emerging markets increased dramatically and commercial bank debt that was the dominant source of foreign capital has been replaced by portfolio flows[4] or foreign direct investment [Bekaert and Harvey (2003)].

EM are today considered as an asset class *per se* by many investors. Emerging economies have passed an important stress test during the period 2008-2009 and are now the key drivers for global growth of the world economy. As pointed out by the JP Morgan recent study[5], *"Potential growth rates for emerging economies of 5.8% now overshadow potential growth of only 1.6% for advanced economies"*. This explains why these markets are associated with very interesting investment opportunities for any investor seeking both returns enhancement and diversification. Inflows into EM have reached a record of US$70 billion in 2010 and will continue to grow as EM yields stay attractive in the context of current global bond markets. Also interesting to notice, the proportion of EM sovereign debt in local currency now accounts for around 80% of the total EM sovereign debt. As a consequence, any simple mean-variance portfolio optimization suggests a high allocation to EM debt. Different client surveys made by banks show an increase in EM debt allocation from around 20% in 2009 to around 25% one year later. Therefore, EM

[3] World Bank define *low GDP* as less than 755 USD per capita.

[4] Essentially composed of fixed income and equity.

[5] JP Morgan Securities, Emerging Markets Research, EM Moves into the Mainstream as an Asset Class, November 23, 2010.

investments appear as really interesting but they suffer from additional risks, such as liquidity risk and, in some cases, contagion effects, that are not taken into account in the basic mean-variance approach or more generally, by asset pricing models. For example, Brandon and Wang (2012) show that the performances of hedge funds are strongly impacted by the consideration of liquidity risk. This is especially the case for hedge funds invested in the asset class of EM. In this paper, we consider the example of index funds invested on Emerging Sovereign Debt Markets.

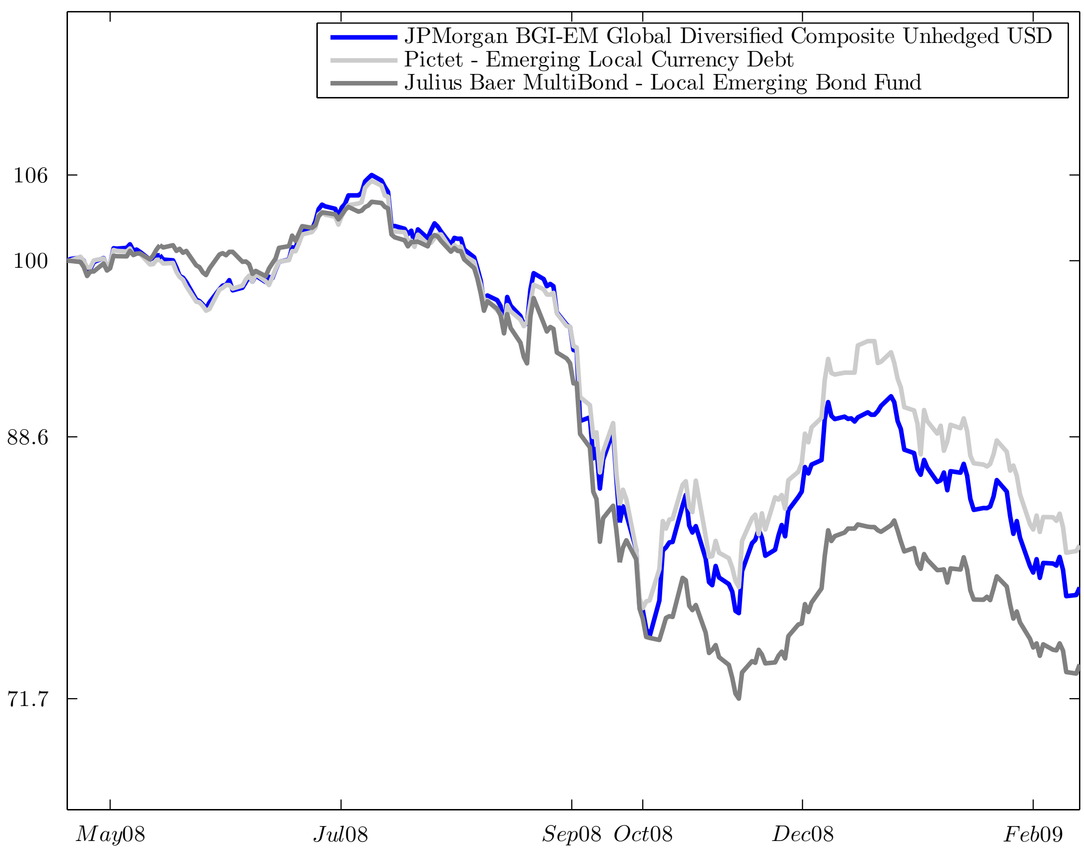

Figure 1 – Both index and funds are presented in USD at a daily frequency from May 2008 to February 2009.

Taking the example of a fund manager tracking the performances of EM, Figure 1 shows that a funding liquidity problem can largely impact the returns of a fund. Indeed, we see that the performances of two different index funds tracking the JP Morgan BGI-EM Index largely differ in October 2008. Pictet (light gray line) has experienced a 800 millions cash outflow corresponding to 10% of the Assets Under Management (AUM) while Julius Baer (dark gray line) did acknowledged a 1400 millions cash outflow representing more than 30% of the AUM.

As we see, the latter exposes the difficulty for the managers tracking the index while exposed to liquidity problems.

2.2 The description of the Sovereign Debt Markets

We use data on sovereign bond yield spreads, sovereign CDS, interest rates and foreign exchange rates[6] for 9 emerging markets: Brazil, Chile, Hungary, Mexico, Poland, Russia, South Africa, Thailand and Turkey. Our sample period is ranging from 01/01/2007 to 26/03/2012. The database is downloaded from Bloomberg.

The time series cover many of the recent crisis and allows us to explore emerging markets behavior during economic disturbances. We are interested to know if it is still possible to benefit from diversification principle and when these benefits could be higher. The CDS premiums are based on 5-year U.S. dollar contracts, for senior claims, and they assume a recovery rate of 25%. We use as risk-free rate the US Swap rate 30/360 paid semiannually.

	Mean	Std	Min	Med	Max	Skewness	Kurtosis
Brazil	148.53	77.94	61.50	123.00	586.41	2.185	7.761
Chile	86.93	59.28	12.50	74.79	322.96	1.369	4.904
Hungary	261.24	172.27	17.34	250.58	738.60	0.314	2.265
Mexico	141.38	90.31	28.17	119.92	601.21	1.755	6.466
Poland	129.56	88.83	7.67	133.50	415.00	0.507	2.809
Russia	209.91	181.84	36.88	153.60	1113.38	2.229	7.758
South Africa	164.00	103.12	24.57	147.80	654.96	1.538	5.891
Thailand	123.30	66.17	31.84	113.01	489.56	1.267	5.242
Turkey	230.46	94.12	118.61	195.72	824.61	2.006	8.447

Table 1 – Summary statistics for CDS premiums from 1-1-2007 to 3-26-2012. Results are expressed in basis points and percentage.

Table 1 provides descriptive statistics for the sovereign CDS premiums, in other words, for the sovereign default risk. The wide range of averages highlights the high degree of heterogeneity among countries with a minimum of 86.93 for Chile and a maximum of 356.95 for Thailand. This is confirmed by the relative STD, where its value for Russia (86.63%) is more than twice that of Turkey (40.84%). For example, the cost of credit protection for Russia increases from 36.88 to 1,113.38 basis points while it reaches only from 12.50 to 322.96 for Chile. We have similar results for the sovereign debt market as shown in Table 2. Brazil has the higher mean with 1242.57

[6] Exchange rates are only used to deal with the problem of different currency issuance among CDS and bonds.

	Mean	Std	Min	Med	Max	Skewness	Kurtosis
Brazil	1242.57	125.59	1000.20	1228.70	1900.40	1.774	8.115
Chile	575.67	80.68	349.96	570.59	822.91	0.373	3.654
Hungary	793.59	134.02	580.50	739.75	1300.03	1.394	4.647
Mexico	704.26	101.70	489.81	731.01	1013.84	-0.228	2.484
Poland	557.46	40.86	474.12	554.89	737.21	0.585	3.383
Russia	777.86	196.81	596.54	712.10	1350.85	1.512	4.126
South Africa	816.56	77.10	679.46	802.22	1109.09	1.001	4.320
Thailand	356.95	65.38	210.32	348.46	571.08	0.515	3.494
Turkey	542.31	146.12	256.20	556.91	1303.43	1.098	6.012

Table 2 – Summary statistics for Bond yields from 1-1-2007 to 3-26-2012. Results are expressed in basis points and percentage.

while Thailand exhibits the lowest average bond yield. The return on an investment into the sovereign debt of one of these countries allowed the investor to earn at least 2.56% annually, investing in Turkey and a maximum of 19% investing in Brazil. Comparing Table 1 and Table 2, we see that the ranking of countries widely differs. Brazil has the biggest bond yield while it is only the sixth country in terms of CDS premium. We have similar results for some other countries that exhibit different behavior of their bond yield relatively to their CDS premium. These tables show that the sovereign debt market is less volatile than the CDS market with a maximum of 26.94% for the relative standard deviation while the minimum for the CDS market is 40.84%.

3 The Emerging Sovereign Debt Market Liquidity measure

In this section, we explore the CDS Bond Spread Basis and discuss its ability to accurately measure the liquidity of the sovereign debt market. Although some other factors in addition to liquidity contribute to the level of the Basis, we explain why they do not have an impact on the dynamic of the liquidity indicator and especially in the case of contagion study. We also present a way to compute the Profit and Loss distribution (P&L hereafter) of CDS and Bonds in order to get them comparable. As they are issued in two different currencies, we can not directly subtract the Bond yield issued in local currency from the CDS Premium expressed in US dollar.

3.1 The CDS Bond Spread basis

CDS were created in 1994 by J.P Morgan & CO. Since its creation the CDS market rose until 2008 and has stagnated since. CDS became in a few years a standardized financial product used by most of the market major participants (banks, hedge funds, mutual funds...). Nowadays, it is one of the most popular tool for transferring credit risk. The CDS contract is defined as a bilateral contract that provides protection on the par value of a specified reference asset. The protection buyer pays a periodic fixed fee or a one-off premium to a protection seller. In return, the seller will make a payment on the occurrence of a specified credit event [Choudhry (2006), Mengle (2007)]. Then, CDS provides to buyer a protection against the risk of default by borrowers, named the entities. The default, also named *credit event* is contractually defined by the two parties and could be bankruptcy, failure to make a schedule payment, obligation default, debt moratorium, financial or debt restructuring and credit downgrade[7]. This is important to precise that rating agencies have not influence in triggering CDS. Their actions may, but not need, taken into account. The protection buyer has to pay an amount of fees (also named CDS premium or CDS spread) to protection seller and receives a payoff if the underlying bond experiences a credit event. At the deal inception, the two parts define which kind of settlement they want. The CDS contract could be settled in one of two ways: cash or physical settlement. Most of the time, contracts are physically settled (about 75-85%). Although the CDS contract has a given maturity, it may terminate earlier if a credit event occurs. In this case, the protection seller has to pay an amount called the protection leg.

The basis is nothing else but correcting the CDS from the sovereign bond (CDS bond spread basis). This is a way to cancel out the global macro effects when analyzing the commonality of sovereign risk. In other words, we focus on the long term liquidity. The basis is defined as the difference between the asset and its synthetic version. The no arbitrage theory of pricing CDS implies that the basis should be zero. As both of these two assets should price the same default risk of the country, from the law of one price, they should be equal. In practice, this situation almost never occurs. The breaking case highlights a liquidity problem on one or the other market. In addition to the liquidity, the level of the basis could fluctuate for many reasons

[7]The main part of CDS are documented using the 2003 ISDA Credit Derivatives Definitions, as supplemented by the July 2009 Supplement.

that could be split into two categories: technical and market factors. We mainly find in the technical factors the delivery option and counterparty risk. To characterize the first, we have to define what *deliverable options* means. CDS contracts usually allow buyer and seller to agree on a panel of alternative assets that the buyer can deliver in case of a credit event. It allows to the buyer to deliver the cheapest obligation that he possesses in his eligible basket of assets. This option does not add value systematically even in the case of sovereign debt market. As we see in Ammer and Cai (2007), the Cheapest-to-Deliver (CtD) option could be valuable for the emerging sovereign debt market. However, our model is based on the existence of frictions interfering with exact arbitrage between CDS and bonds. One of these frictions we are particularly interested in is the liquidity of the sovereign debt market. In this context, it becomes really difficult to model and evaluate the CtD option. Indeed, Ammer and Cai (2007) propose to measure the spread part that could be attributed to the CtD option. Their model requires two strong assumptions allowing to measure the CtD option: the recovery rate is independent of time-to-default and the CtD option is the only friction. This second assumption is not realistic in our case and this is empirically proved that market liquidity is one of the main frictions interfering in the arbitrage relation between the CDS premium and the bond yield spread over the risk free rate. As the CtD option, although valuable, is sometimes null we neglect it in our model to focus on the market liquidity. The second is the counterparty risk. On the one hand, the protection seller can default and do not settle the protection buyer in case of a credit event. On the other hand, the buyer can also default and stop paying the CDS premium to the seller. However, some mechanisms like the counterparty clearing system allow to reduce these risks (almost half of CDS are treated by clearing). Moreover, as showing in Levy (2009), if the default probability of the underlying bond and the default probability of the counterparty are not correlated, the two effects may cancel each other out. Furthermore, counterparty risk is a joint event of two defaults. Thus, the excess premium associated is weighted by a product of two probabilities and should be really small, or negligible. Our aim being the analysis of the dynamic of the emerging market liquidity, we consider that the main part of CDS is issued by companies in countries which are outside our sample of EM. As a consequence, if the counterparty risk changes, its impact is approximately the same for all countries and does not alter the dynamic of correlations that we study. Based on a demonstration proposed by Levy (2009), we focus on the liquidity premium induced by the movements of the basis on emerging markets.

CDS includes two legs corresponding to the premium payments and the default payment. The pricing of a CDS depends, among others, on the recovery amount (a recovery rate of par value and accrued interest). Duffie (1999) or Hull and White (2000) expose two approaches for the pricing of CDS premium. The first, that we call ”no arbitrage” approach, follows the idea that an investor can buy a CDS and the underlying bond to replicate the risk free rate. The second is based on a reduced-form model with random stopping time. In order to demonstrate the impact of liquidity on the CDS Bond spread basis, we use the first one. As a result, buying a risky bond and its CDS with the same maturity allow to the investor to eliminate the default risk associated with the bond. Assuming that there is no arbitrage opportunity, this portfolio should be equal to the value of the risk free bond with the same maturity. As in Zhu (2006), we price CDS premiums and Bonds separately. We construct a portfolio that replicate the CDS contract and we obtain the CDS Spread Basis. In this context, we assume a risk neutral world with three assets: a risk-free bond, a risky bond and a CDS contract.

Following Levy (2009), under the risk neutral valuation, we express the value of a CDS premium, a risky bond and a risk-free bond. Then, we construct a portfolio that shorts the risky bond and buys the risk free bond subtracting (4) to (5). We propose details of computations in appendix and finally, assuming that the risky bond is traded at par, thus we have:

$$b - (y - r) = 0 \tag{1}$$

As a consequence, the CDS Bond Spread Basis is equal to zero, theoretically. However, this relation changes assuming that there are two types of traders: one trading with high liquidity and no holding costs (h) and the other one trading with low liquidity and having holding cost of d (l).

In this case, both the CDS premium and the Bond yield have an additionnal component due to the search cost of liquidity. As a result, we have:

$$\tilde{b} = \tilde{y} - r - (S_{bond} - S_{CDS}). \tag{2}$$

Thus, the parity between CDS and risky bond should hold only for the pure risk component that is priced into the two assets. As a consequence, we can expect a non zero basis when liquidity differences exist.

3.2 Basis trade with multiple currencies

The CDS Bond Spread basis that compares CDS premium denominated in dollar to the local currency denominated sovereign debt is biased. To tackle this problem, we compute and correct the P&L of an investment strategy corresponding to the basis. In other words, we buy/sell both instruments when the basis is negative/positive. The computation of the P&L is the way that traders refer to the daily change of the value of their trading positions. The P&L is generally defined as the difference between the value at time $t + 1$ and t. In other words, the P&L of an asset is the profit or the loss that this asset makes between two dates. In this sense, we can split the P&L between two parts: the Mark-to-Market (MtM hereafter) part and the Carry part. The former is the gain (or the loss) realized when selling the asset. The latter, called the carry, is the gain (or the loss) i.e., the income you earn on the asset during the period you own it (in our case, one day). Schematically, we can express the P&L of an asset as the sum of the MtM P&L and the Carry P&L.

Both CDS and Generic bonds P&L are computed at a daily frequency, and thus, ignoring the carry. This component is very close to zero due to the daily investment horizon. As a consequence, the annual return of the asset has to be divided by 250, but could be neglected in the P&L computation. We detail computation of P&L for both CDS and Generic Bond in appendix B. Once we compute these time series of P&L, we can easily calculate the CDS Bond Spread Basis. Indeed, when the basis is negative, we buy both the CDS protection and the bond and conversely when the basis is positive. As a result, in the first case, we add the two P&L of the CDS and the bond while we make the sum of their opposite value in the second case.

Our empirical study of the market liquidity indicators confirms some stylized facts and the collapse of Lehman Brothers just as the 2007-2008 financial crisis are strongly highlighted. Figure 2 presents, for each of the 9 emerging countries of our sample, the level of CDS Bond spread basis. Firstly, we can see that the basis is almost never equal to zero indicating that the arbitrage relation is not verified. We see that all countries experienced almost simultaneously a liquidity problem. Indeed, the graphics reveal a large increase of the basis for every country at the end of September 2008. Even if the close relation between all the basis is obvious during this period, the increase of volatility for all the markets may be the only source of contagion. If this is true, the fund manager does not have to change his portfolio allocation. However, if the correlations increase despite a control for volatility, corresponding to pure contagion effects, the fund manager is exposed to a new risk and the allocation of his portfolio is not efficient anymore.

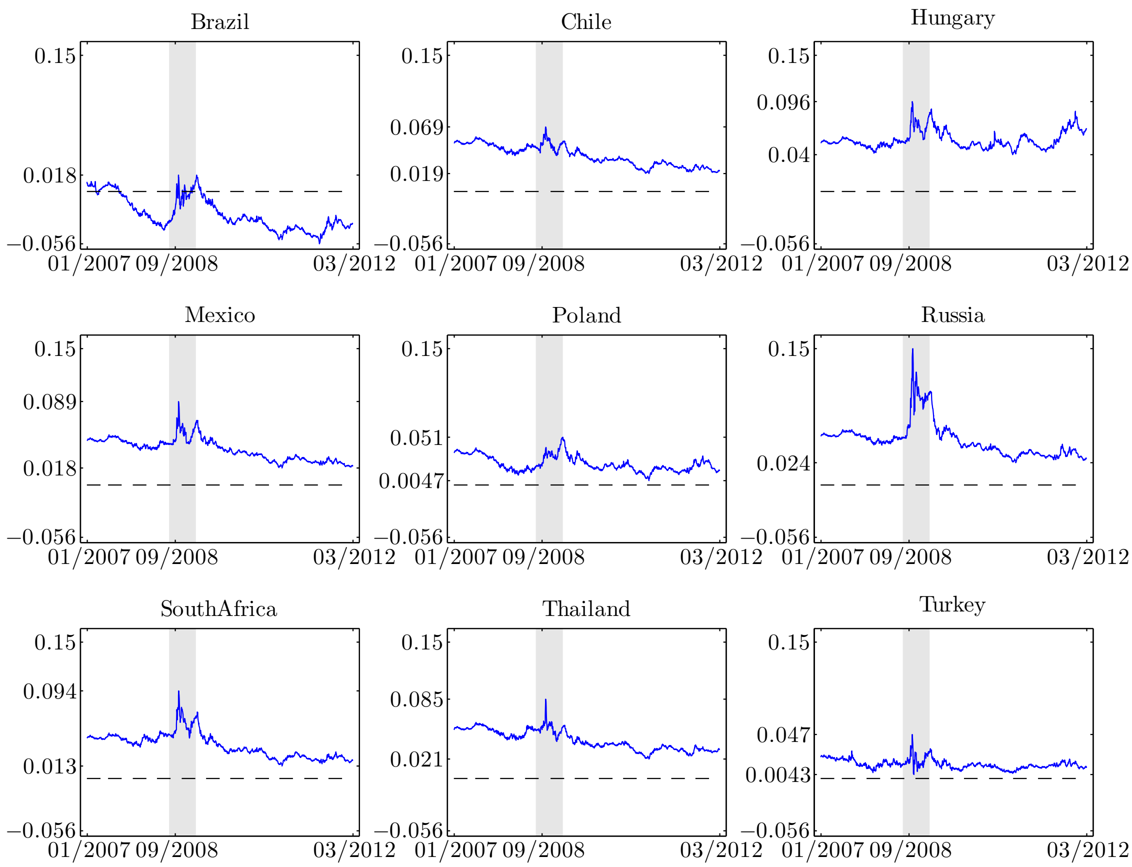

Figure 2 – CDS Bond spread basis for each of the 9 emerging countries from the 01/01/2007 to the 03/26/2012. The results are computed with daily observations, expressed or corrected in US dollars. The grey band represents a period with explicit higher correlations between all the basis.

Finally, coming back to liquidity, we know that the contagion between markets drives their in and outflows, and liquidity moves consequently. In the line of the above approach, we can link contagion and liquidity moves by comparing the commonalities between the liquidity indicators introduced in the previous section. If the commonality is between liquidity and volatility, there is no contagion effect but only interdependence. On the contrary, if the liquidity shock has an impact on the correlation matrix, liquidity can be considered as a contagion channel.

4 Liquidity contagion and funding liquidity

Now, we focus on the dynamic of the link between market liquidity risks of sovereign debt markets. In order to detect financial contagion effects and following Pelletier (2006), we apply the RSDC model. This latter is a dynamic model allowing a regime switching of the correlation matrix and a conditional modeling of volatility. These two characteristics are very relevant when we focus on financial contagion effects. Firstly, we have to make the distinction between interdependence and pure contagion. On the one hand, the first is represented by common exogenous shocks able to increase links between countries or markets without a significant change of dependence. On the other hand, the pure contagion is described as a significant increase of correlations after a shock and considering the previous link between markets or countries. We find a large literature that focuses on the link between an increase of the volatility and an increase of correlations. For example, Forbes and Chinn (2004) show that correlations between countries go up in crash times. But, several papers demonstrate that a positive evolution of the volatility leads to an increase of correlations (e.g. Hamilton and Susmel (1994), Solnik et al. (1996) or Chesnay and Jondeau (2001)). Secondly, in order to detect and identify pure contagion effects, the definition of crisis periods has to be endogenous. The RSDC is a state/space model allowing to specify the dates of crisis in an endogenous way. As a result, we are able to detect if there exists a shift in terms of correlations and we also know the period when this contagion effects occurs. The RSDC model is detailed in as one of the most performing to model the behavior of correlations compared to other multivariate conditional correlation models. Finally, we choose a two-regimes RSDC since we only need to detect a shift in terms of correlations, represented here as low or high.

4.1 Filtering a common liquidity factor

Usually, we test for financial contagion considering the price as the main channel. However, in this paper, we focus on the contagion in terms of liquidity. As a result, we filter a common liquidity factor from several market liquidity indicators. Indeed, we study the smoothed probabilities obtained using the RSDC model. This latter is nothing but the probability to be in a state for which the correlations are high relatively to other periods. Indeed, as we focus on detecting shift in terms of correlations, we only consider two states for the RSDC model. The first is a state of calm periods and the second is a state of crisis periods characterized by higher correlations. Figure 3 displays the smoothed probability obtained based on the CDS Bond Spread basis of 9 emerging countries. We see that the probability to be highly correlated is very low, almost null; with some peaks from the start of our sample period to October 2008. Thus, we can conclude that the Lehman Brother collapse has a great influence on the re-correlation phenomenon. The important fact is that the probability stays high after the end of 2008 meaning that we do not come back to a normal state even after some years.

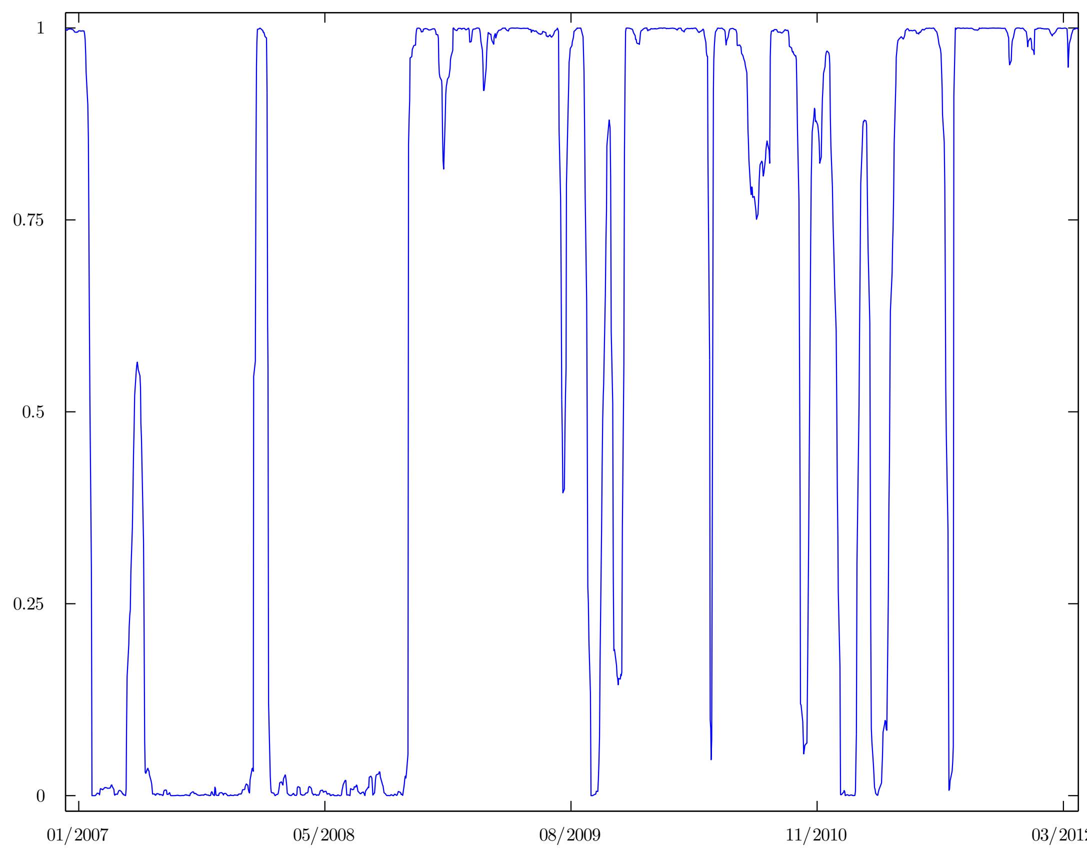

Figure 3 – Smoothed Probabilities to be in the state of high correlations at a daily frequency. The results concern the basis between 01/01/2007 and 26/03/2012.

To confirm the results of pure contagion effects, we have to determine whether the two correlation matrices are significantly different. In other words, we test the number of regimes.

However, the Markov switching approach does not allow us to apply standard tests methods. Under the null hypothesis, a nuisance parameter is not identified. Garcia (1998) shows that asymptotic theory works for Markov switching only assuming the validity of the score distribution. Nevertheless, the asymptotic distribution is not so far from the standard Chi-square distribution while our likelihood ratio statistic is much greater than the critical value of this distribution. We conclude that a two regimes model offers greater results and confirm the significance of the difference between the two correlation matrices. To compute this statistic, we have to compare the likelihood of our model, the RSDC with two regimes and the CCC model (for Constant Conditional Correlations) which is assimilated to the RSDC with only one regime.

	Brazil	Chile	Hungary	Mexico	Poland	Russia	South Africa	Thailand	Turkey
Brazil		-0.015	-0.092	-0.049	0.317	-0.059	0.259	-0.009	0.290
Chile	-0.087		0.284	0.393	0.043	0.345	0.207	0.314	-0.178
Hungary	-0.052	0.422		0.364	-0.043	0.439	0.301	0.244	-0.291
Mexico	0.067	0.612	0.484		-0.063	0.647	0.222	0.289	-0.277
Poland	0.337	-0.063	0.099	-0.024		-0.134	0.268	-0.025	0.382
Russia	-0.087	0.493	0.753	0.515	0.033		0.331	0.376	-0.371
South Africa	0.148	0.274	0.569	0.352	0.332	0.641		0.157	0.070
Thailand	-0.021	0.317	0.446	0.361	0.090	0.486	0.384		-0.239
Turkey	0.381	-0.192	-0.052	-0.133	0.512	-0.085	0.228	-0.076	

Table 3 – Correlations matrices of the two regimes for the basis considered in terms of profits and losses generated by a such strategy. The upper part of the matrix corresponds to the regime 1 while the lower part corresponds to the regime 2.

Table 3 presents the values of correlations both for state 1 and state 2 based on the CDS Bond Spread basis. The RSDC model has the main advantage to propose an aggregate result for a multivariate model. We see that almost all correlation pairs increase going from regime 1 to regime 2. As a consequence, in a general way, we can conclude that the links between countries become stronger in the state considered as defining turmoil periods.

4.2 Identifying as a funding liquidity factor

The funding liquidity is still difficult to be measured. However, Goyenko (2012) among others[8] shows that the TED spread and the VIX could be considered as funding liquidity indicators. In this subsection, we study the link between our Funding Liquidity Indicator (or FLI hereafter) and the TED spread or the VIX.

[8] Brunnermeier and Pedersen (2009), Boyson et al. (2010) or Teo (2011) describe the TED spread and the VIX as an indicator of speculator's capital availability in the economy.

The TED spread is the difference between the three-month LIBOR and the three-month T-bill interest rate. It reflects the credit risk of the financial system as a whole. The VIX is a measure of the implied volatility of S&P 500 index options. It denotes the predicted volatility over the next month and it is usually defined as a measure of fear on financial markets. Figure 4 displays the behavior of the TED spread and the VIX compared to the dynamic of the FLI. We show some similarities between the TED spread and the VIX with a stronger volatility in the case of the VIX. However, both of them have the same behavior at the end of 2008. All the three indicators experience a large increase during October 2008. However, their dynamic largely differ after this event since both the TED spread and the VIX indicate a come back to a normal state while the FLI remains high indicating that funding liquidity problems still occurs.

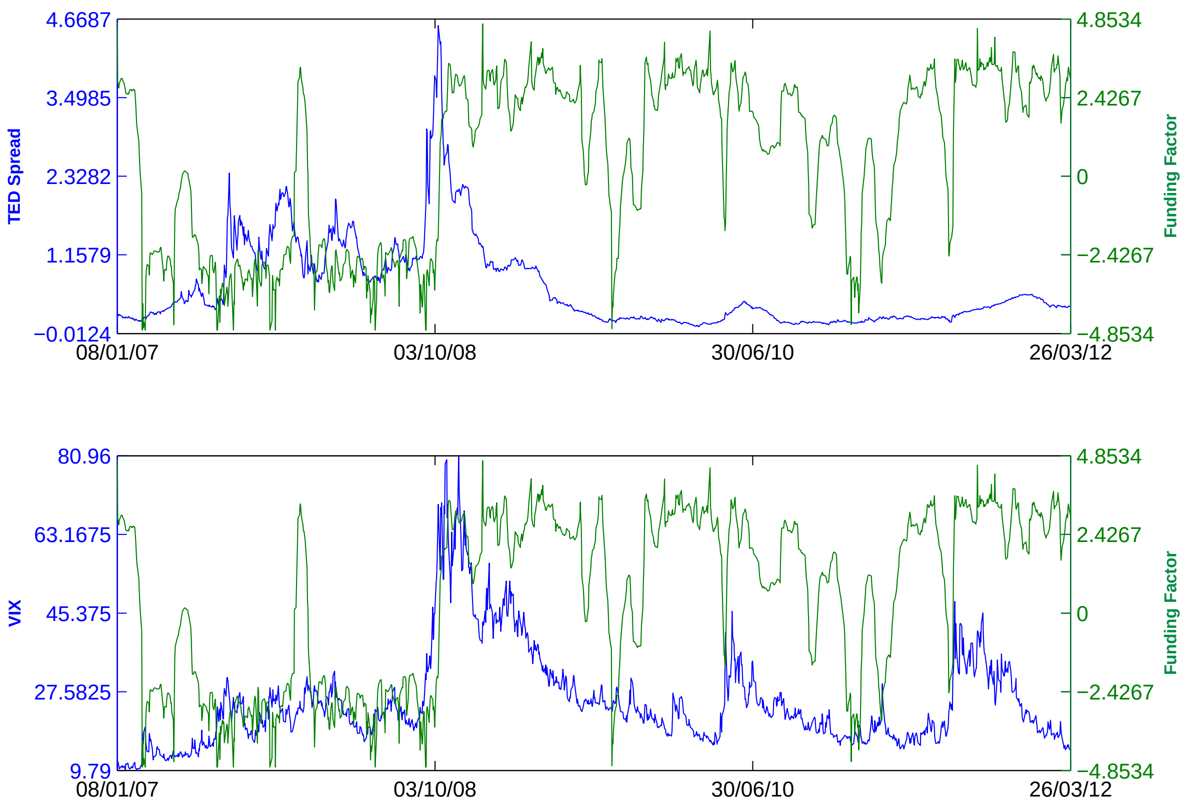

Figure 4 – The top graph presents TED spread and our funding liquidity indicator (FLI) while the bottom graph displays the VIX and the FLI. Both of them are at a daily frequency ranging from 01/01/2007 to 26/03/2012.

	Before 2009			After 2009			Whole sample		
Intercept	0.158**	-0.077**	-0.044	0.701**	0.602**	0.682**	0.737**	0.310**	0.212**
	(4.976)	(-2.396)	(-1.492)	(39.039)	(17.595)	(18.597)	(42.902)	(10.198)	(8.508)
TED	0.057**		-0.275	0.373**		0.345**	-0.205**		-0.474**
	(2.557)		(-1.492)	(9.045)		(5.466)	(-11.218)		(-25.979)
VIX		0.012**	0.024**		0.009**	0.001		0.011**	0.028**
		(10.844)	(14.764)		(7.096)	(0.578)		(10.419)	(25.512)
R^2	1.33%	19.45%	31.88%	9.37%	5.99%	9.41%	8.95%	7.82%	39.66%

Table 4 – This table presents results of time series regressions from 01/01/2007 to 31/12/2012. We split the sample in two parts, before and after 2009. The betas are estimated as the slope coefficients of our funding liquidity indicator (or FLI) on intercept, TED spread and VIX. The t-statistics are in parentheses.
* Denotes parameter estimates significant at 10% level.
** Denotes parameter estimates significant at 5% level.

Table 4 presents the regression results of the FLI on the TED spread and the VIX. We see the link between them is statistically significant whatever the sample studied. Indeed, except for the TED spread before 2009 and the VIX after 2009 that are not significant at 10% when we consider it simultaneously with the other indicator, all the estimated betas are significant at a 5% threshold. As we perceive on Figure 4, the relation between the FLI and the VIX is stronger before 2009 than after. Moreover, the volatility indicator has a positive impact on the funding liquidity indicator in all cases while the TED spread has an opposite impact after 2009 and over the whole sample. Finally, we see that the R^2 is largely greater when we consider the VIX rather than the TED spread before 2009. Moreover, it raises from 19% to 32% when considering both of them. Nevertheless, when we focus on the results after 2009, we see that the explanation capacity of the two indicators is really limited. The R^2 remains under 10% even considering both TED and VIX. All these results indicate that the FLI, a common liquidity factor is able to measure funding liquidity problems. Moreover, while the usual indicators appear driven by other factors in addition to funding liquidity, the FLI adopts a different behavior still indicating funding liquidity problems until the end of the sample.

4.3 Pricing the impact of funding liquidity

	CAPM	Fama French	TED Spread	CAPM	Fama French
				and TED Spread	
Intercept	0.0005*	0.0001	-0.0001	0.0004	0.0001
	(1.7601)	(0.5928)	(-0.2111)	(1.0505)	(0.3077)
Market	-0.0005**	-0.0007**		-0.0003	-0.0005
	(-43.617)	(-4.3824)		(-0.5467)	(-1.2951)
HmL		0.0000			0.0000
		(-0.4334)			(-0.0138)
SmB		-0.0003**			-0.0002
		(-4.2200)			(-1.2417)
TED Spread			-0.0001	0.0003	0.0003*
			(-0.1684)	(1.1814)	(1.7017)
R2	7.57%	9.13%	5.33%	11.98%	13.58%

Table 5 – **Before 2009** This table presents results of time series regressions and shows average coefficients through 112 index funds invested in EM. All the columns present estimated betas as the slope coefficient in regressions of excess returns on the the CAPM, the Fama French three factors and the TED spread. The second panel present results adding the TED spread in each model. The *t*-statistics are robusts (Newey-West) and presented between parenthesis.
* Denotes parameter estimates significant at 10% level.
** Denotes parameter estimates significant at 5% level.

	CAPM	Fama French	FLI	CAPM	Fama French
				and FLI	
Intercept	0.0005*	0.0001	0.0000	0.0005**	0.0002*
	(1.7601)	(0.5928)	(-0.0615)	(2.6161)	(1.6622)
Market	-0.0005**	-0.0007**		-0.0010**	-0.0010**
	(-43.617)	(-4.3824)		(-3.1035)	(-2.9281)
HmL		0.0000			0.0001
		(-0.4334)			(0.7553)
SmB		-0.0003**			-0.0004**
		(-4.2200)			(-2.8754)
FLI			-0.0012**	0.0005**	0.0004**
			(-3.1571)	(2.5178)	(2.7576)
R2	7.57%	9.13%	0.40%	7.97%	9.53%

Table 6 – **Before 2009** This table presents results of time series regressions and shows average coefficients through 112 index funds invested in EM. All the columns present estimated betas as the slope coefficient in regressions of excess returns on the the CAPM, the Fama French three factors and the FLI. The second panel present results adding the FLI in each model. The *t*-statistics are robusts (Newey-West) and presented between parenthesis.
* Denotes parameter estimates significant at 10% level.
** Denotes parameter estimates significant at 5% level.

	CAPM	Fama French	TED Spread	CAPM	Fama French
				and TED Spread	
Intercept	0.0006**	0.0003**	0.0006**	0.0007**	0.0004**
	(11.015)	(3.9259)	(3.3500)	(4.0478)	(3.3863)
Market	0.0002**	0.0000		0.0001	-0.0001
	(4.0316)	(-0.1917)		(0.7010)	(-0.5751)
HmL		0.0004**			0.0002**
		(3.1024)			(2.0315)
SmB		-0.0002			-0.0003
		(-1.1617)			(-1.3204)
TED Spread			0.0001	0.0002	0.0003**
			(0.7132)	(1.4406)	(11.2208)
R2	7.90%	8.44%	0.52%	8.42%	9.00%

Table 7 – **After 2009**This table presents results of time series regressions and shows average coefficients through 112 index funds invested in EM. All the columns present estimated betas as the slope coefficient in regressions of excess returns on the the CAPM, the Fama French three factors and the TED spread. The second panel present results adding the TED spread in each model. The t-statistics are robusts (Newey-West) and presented between parenthesis.
* Denotes parameter estimates significant at 10% level.
** Denotes parameter estimates significant at 5% level.

	CAPM	Fama French	FLI	CAPM	Fama French
				and FLI	
Intercept	0.0006**	0.0003**	0.0002**	0.0004**	0.0002**
	(11.015)	(3.9259)	(2.6726)	(3.9258)	(2.3883)
Market	0.0002**	0.0000		0.0001	-0.0001
	(4.0316)	(-0.1917)		(0.2814)	(-0.1917)
HmL		0.0004**			0.0003
		(3.1024)			(0.9927)
SmB		-0.0002			-0.0002
		(-1.1617)			(-1.1908)
FLI			0.0001	0.0003	0.0003**
			(0.2889)	(0.8697)	(3.1572)
R2	7.90%	8.44%	0.04%	7.95%	8.49%

Table 8 – **After 2009** This table presents results of time series regressions and shows average coefficients through 112 index funds invested in EM. All the columns present estimated betas as the slope coefficient in regressions of excess returns on the the CAPM, the Fama French three factors and the FLI. The second panel present results adding the FLI in each model. The t-statistics are robusts (Newey-West) and presented between parenthesis.
* Denotes parameter estimates significant at 10% level.
** Denotes parameter estimates significant at 5% level.

5 Robustness Checks

5.1 The heteroscedasticity problem

All the results are based on the RSDC model of Pelletier (2006) and its capacity to distinguish interdependence from pure contagion. We consider a two-step estimation and assume that the first one is able to correct for heteroscedasticity. In this robustness check, we want to control whether the volatility is removed accurately allowing to concentrate on pure contagion phenomena. In other words, the univariate conditional volatility model where the matrix S_t in equation (28) comes from all the dynamic of the univariate variance. We check for ARCH effects in the standardized residuals and determine the best model for the first step in the estimation. We start with the well known GARCH(1,1) model, and when we reject the hypothesis of independence of the squared residuals, we estimate a TGARCH(1,1) taking into account the asymmetry of P&L time series. When there is still some remaining heteroscedasticity in all time series, we propose an alternative model [TGARCH(2,2)].

We sum up the results in Table 11. First, we see that the GARCH(1,1) model captures all heteroscedasticity effects for all countries except for Hungary in the case of the CDS and the Basis. However, in a second step, we see that even a TGARCH(1,1) cannot capture the remaining ARCH effects. In fact, we have to apply a TGARCH(2,2) on our time series to get homoscedastic standardized residuals. To test if the univariate volatility estimation fully captures the heteroscedasticity, we use the LM test [Engle (1982)] that checks for the autocorrelation of squared residuals. We can conclude about the presence of ARCH effects in the residuals of our model.

However, whatever the univariate volatility model, we detect pure contagion effects at the very same date. Indeed, the smoothed probabilities have a similar dynamic no matter which model is applied. As a result, the RSDC model is robust to the volatility specification and allows us to concentrate on pure contagion effects resulting from a shift in the correlation structure.

5.2 Pure contagion in terms of prices

Usually, the contagion is studied in terms of prices but we show in this paper that the liquidity also experiences pure contagion effects. It appears relevant to determine whether there exists an additional information focusing on the liquidity contagion. Thus, we propose to apply our financial contagion model to the CDS premium and the Bond yield.

We give in Figure 5 the smoothed probabilities of being in the regime of high correlation for the bond market. We see that the correlations appear to be dynamic and switch between regimes. At each date, there is an uncertainty about the regime of correlations. On the one hand, the process spends more time in regime two and spells in regime one are shorter on average. The probability of being in regime two at time t, conditional on being in regime two at time $t-1$, $p_{2,2}$ is 0.9998. Such a high probability means a very high level of persistence in the Markov chain. In comparison, for regime one this probability fells around 0.9040. Despite the close proximity of these probabilities, as Pelletier (2006) shows, it results in large differences. Indeed, after 5 periods, these probabilities are respectively approximately equal to 0.95 and 0.55[9]. On the other hand, we show that the magnitudes of almost all the correlations in regime one are smaller than in regime two. Correlation matrices are presented in appendix E page 40. Moreover, the smoothed probability to be in state two largely increases at the end of September 2008. As a consequence, we can consider that there exists a re-correlation phenomenon on the sovereign debt market and this phenomenon occurs almost simultaneously with the Lehman Brother collapse.

The results of the CDS market are similar (see Figure 6). Indeed, the shift in correlations appears at the same date as the bond market. However, the smoothed probability to be in the state of high correlations for the CDS market is more volatile than for the bond market. The probability to be in the state of low correlations at time t and $t+1$ is equal to 0.9414 which is much greater than in the case of the bond market. Thus, as both CDS and bond markets exhibit shifts in terms of correlations at the end of 2008, the benefits from the diversification principle have plummet and the risk of the portfolios dramatically increased. Furthermore, we have to know if this phenomenon also comes from a liquidity problem since it would represent an additional risk for the fund manager. Moreover, we state that even if the assets behaviors come back to normal after some months, the correlations between them stay high. In that case, it remains particularly difficult to benefit from the diversification principle on the sovereign debt markets.

[9] Indeed, $0.99^5 = 0.95$ and $0.90^5 = 0.55$

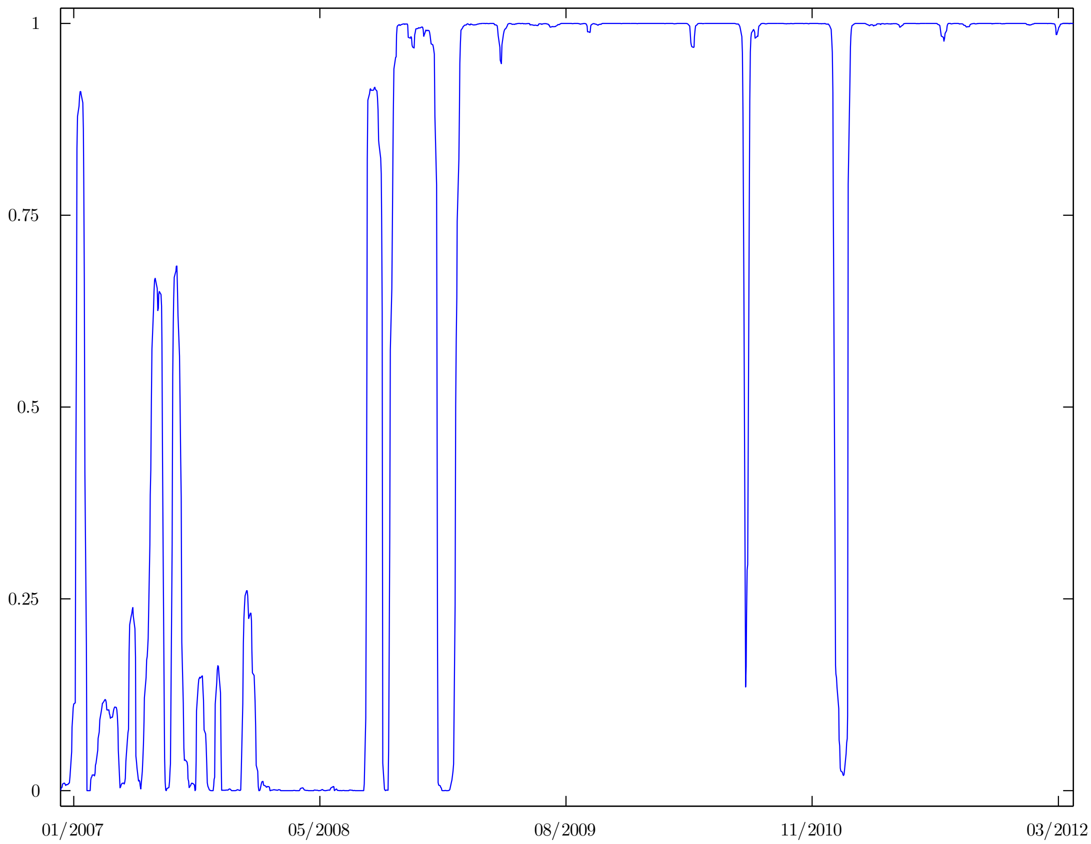

Figure 5 – Smoothed Probabilities to be in the state of high correlations at a daily frequency. The results concern the bond market between 01/01/2007 and 26/03/2012.

Finally, the only common event is the shift that appears at the end of 2008. However, we show that focusing on market liquidity indicators leads to additional information about the behavior of emerging sovereign debt markets and allows a better understanding of the funding liquidity risk.

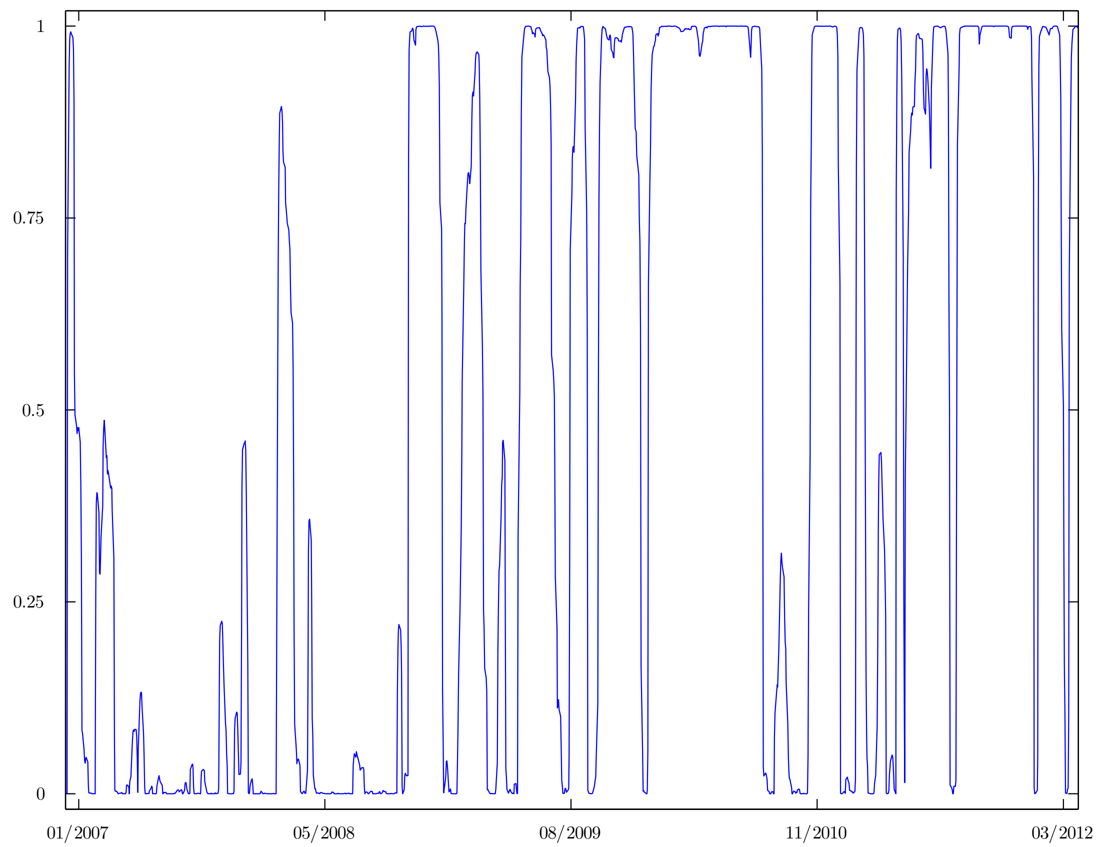

Figure 6 – Smoothed Probabilities to be in the state of high correlations at a daily frequency. The results concern the CDS market between 01/01/2007 and 26/03/2012.

6 Conclusion

EM have experienced many financial crisis with contagion problems but, they are today the key drivers for global growth of the world economy. They propose very attractive investment opportunities for asset managers who consider them as an asset class. Nevertheless, the main risk for an asset manager is to loose the diversification benefits of his/her portfolio.

Firstly in this paper, we study the liquidity of the sovereign debt market showing the ability of the CDS Bond spread basis to accurately measuring liquidity. Secondly, we use a non linear model to detect contagion effects both in terms of prices and liquidity. Whereas interdependence is not a main concern for a fund manager, pure contagion phenomena may be problematic. The RSDC model is able to separate interdependence from pure contagion in order to focus on the second one. Such a phenomenon is highlighted by a shift in the probability to be in the state where the assets are more correlated.

Our main focus is the pure contagion phenomena in terms of market liquidity problems that we detect during the 2007-2008 financial crisis for a sample of 9 emerging countries. Indeed, the contagion model allows us to extract a funding liquidity indicator based on the market liquidity of the sovereign debt of several countries. As a result, we show there still exists a funding liquidity problem on the emerging sovereign debt market while standard indicators let appear a come back to a pre crisis state.

References

Adrian, T. and Shin, H. S. (2008). Liquidity and financial contagion. *Banque de France Financial Stability Review: Special Issue on Liquidity*, 11:1–7.

Aitken, M. and Winn, R. (1997). What is this thing called liquidity. *Securities Industry Research Center of Asia Pacific Sydney, Australia*.

Ammer, J. and Cai, F. (2007). Sovereign CDS and bond pricing dynamics in emerging markets: does the cheapest-to-deliver option matter? *International Finance Discussion Papers*.

Bai, J. and Collin-Dufresne, P. (2011). The determinants of the cds-bond basis during the financial crisis of 2007-2009. *SSRN eLibrary*.

Bekaert, G. and Harvey, C. (2003). Emerging markets finance. *Journal of Empirical Finance*, 10(1-2):3–56.

Bollerslev, T. (1990). Modelling the coherence in short-run nominal exchange rates: a multivariate generalized arch model. *The Review of Economics and Statistics*, pages 498–505.

Boyson, N. M., Stahel, C. W., and Stulz, R. M. (2010). Hedge fund contagion and liquidity shocks. *The Journal of Finance*, 65(5):1789–1816.

Brandon, R. G. and Wang, S. (2012). Liquidity risk, return predictability and hedge funds'performance: an empirical study. *Journal of Financial and Quantitative Analysis, forthcoming*.

Brunnermeier, M. K. and Pedersen, L. H. (2009). Market liquidity and funding liquidity. *Review of Financial Studies*, 22(6):2201–2238.

Chesnay, F. and Jondeau, E. (2001). Does correlation between stock returns really increase during turbulent periods? *Economic Notes*, 30(1):53–80.

Choudhry, M. (2006). *The credit default swap basis*. Bloomberg Pr.

Dempster, A., Laird, N., and Rubin, D. (1977). Maximum likelihood from incomplete data via the em algorithm. *Journal of the Royal Statistical Society. Series B (Methodological)*, pages 1–38.

Duffie, D. (1999). Credit swap valuation. *Financial Analysts Journal*, 55(1):73–87.

Engle, R. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of united kingdom inflation. *Econometrica: Journal of the Econometric Society*, pages 987–1007.

Engle, R. (2002). Dynamic conditional correlation. *Journal of Business and Economic Statistics*, 20(3):339–350.

Fontana, A. and Scheicher, M. (2010). An analysis of euro area sovereign cds and their relation with government bonds. *SSRN eLibrary*.

Forbes, K. J. and Chinn, M. D. (2004). A decomposition of global linkages in financial markets over time. *Review of Economics and Statistics*, 86(3):705–722.

Garcia, R. (1998). Asymptotic null distribution of the likelihood ratio test in markov switching models. *International Economic Review*, pages 763–788.

Garleanu, N. and Pedersen, L. (2009). Margin-based asset pricing and deviations from the law of one price. *Working Paper, UC Berkeley*.

Giamouridis, D. and Vrontos, I. (2007). Hedge fund portfolio construction: A comparison of static and dynamic approaches. *Journal of Banking & Finance*, 31(1):199–217.

Goyenko, R. (2012). Treasury liquidity and funding liquidity: Evidence from mutual fund returns. *Available at SSRN*.

Goyenko, R., Holden, C., and Trzcinka, C. (2009). Do liquidity measures measure liquidity? *Journal of Financial Economics*, 92(2):153–181.

Gromb, D. and Vayanos, D. (2002). Equilibrium and welfare in markets with financially constrained arbitrageurs. *Journal of Financial Economics*, 66(2):361–407.

Hamilton, J. D. and Susmel, R. (1994). Autoregressive conditional heteroskedasticity and changes in regime. *Journal of Econometrics*, 64(1):307–333.

Hasbrouck, J. (2004). Liquidity in the futures pits: Inferring market dynamics from incomplete data. *Journal of Financial and Quantitative Analysis*, 39(2).

Holmes, R. (1991). On random correlation matrices. *SIAM journal on matrix analysis and applications*, 12:239.

Hull, J. and White, A. (2000). Valuing credit default swaps I: No counterparty default risk. *Journal of Derivatives*, 8(1):29–40.

Lesmond, D., Ogden, J., and Trzcinka, C. (1999). A new estimate of transaction costs. *Review of Financial Studies*, 12(5):1113.

Levy, A. (2009). The cds bond basis spread in emerging markets: Liquidity and counterparty risk effects. Working paper.

Mengle, D. (2007). Credit derivatives: An overview. *Economic Review*, (Q4):1–24.

Menkveld, A. and Wang, T. (2011). Liquileaks.

Morris, S. and Shin, H. (2004). Coordination risk and the price of debt. *European Economic Review*, 48(1):133–153.

Pelletier, D. (2006). Regime switching for dynamic correlations. *Journal of Econometrics*, 131(1-2):445–473.

Roll, R. (1984). A simple implicit measure of the effective bid-ask spread in an efficient market. *Journal of Finance*, 39(4):1127–1139.

Solnik, B., Boucrelle, C., and Le Fur, Y. (1996). International market correlation and volatility. *Financial analysts journal*, pages 17–34.

Teo, M. (2011). The liquidity risk of liquid hedge funds. *Journal of Financial Economics*, 100(1):24–44.

Zakoian, J. (1994). Threshold heteroskedastic models. *Journal of Economic Dynamics and control*, 18(5):931–955.

A The CDS Bond Spread basis

The CDS premium b satisfies:

$$\sum_{t=1}^{T} be^{-rt}F(t) = \int_{0}^{T} (100 - RV_t)e^{-rt}f(t)dt, \quad (3)$$

where T is the number of times till maturity or default, r is the risk-free rate, RV_t is the recovery value at time t, $f(t)$ is the probability of default at time t and $F(t)$ is the survival probability[10]. The left hand side is called the Premium leg and the right hand side is called the Insurance leg.

The value of the risky bond is expressed as:

$$Y = \sum_{t=1}^{T} Ce^{-rt}F(t) + 100e^{-rT}F(T) + \int_{0}^{T} RV_t e^{-rt}f(t)dt, \quad (4)$$

where C is the fixed coupon paid for each period.

And the value of a risk-free bond at risk-free rate r is expressed as:

$$100 = \sum_{t=1}^{T} re^{-rt} + 100e^{-rT}. \quad (5)$$

We construct a portfolio that shorts the risky bond and buys the risk free bond subtracting (4) to (5). We obtain:

$$100 - Y = \sum_{t=1}^{T} re^{-rt} + 100e^{-rT} - \sum_{t=1}^{T} Ce^{-rt}F(t) - 100e^{-rT}F(T) - \int_{0}^{T} RV_t e^{-rt}f(t)dt. \quad (6)$$

If we modify the risk-free bond equation to include the default probability, we get:

$$100 = \sum_{t=1}^{T} re^{-rt}F(t) + 100e^{-rT}F(T) + \int_{0}^{T} 100e^{-rt}f(t)dt. \quad (7)$$

The value of our portfolio becomes:

[10] $F(t) = 1 - \int_0^t f(x)dx$

$$100 - Y = \sum_{t=1}^{T}(r - C)e^{-rt}F(t) + \int_0^T (100 - RV_t)e^{-rt}f(t)dt. \quad (8)$$

Combining equation (3), we can write:

$$100 - Y = \sum_{t=1}^{T}(b + r - C)e^{-rt}F(t) \quad (9)$$

Finally, the CDS Bond Spread Basis is expressed as:

$$b + (r - C) = \frac{100 - Y}{\sum_{t=1}^{T} e^{-rt}F(t)} \quad (10)$$

The CDS Spread Basis has to be equal to zero since the risky bond is traded at par, i.e. $Y = 100$. Moreover, the fixed coupon of a par bond is equal to the bond's yield to maturity $(y = C)$ and we have:

$$b - (y - r) = 0 \quad (11)$$

Furthermore, assuming that there are two types of traders: one trading with high liquidity and no holding costs (h) and the other one trading with low liquidity and having holding cost of d (l). We denote by b_i the CDS premium fair price for trader i, $i = l, h$, $\tilde{S}$, the market price for this CDS and p_i the probability to immediately find a trader of type i. We know that a trader, who has liquidity problems, should pay an additional holding cost. Then, from equation (3) we obtain:

$$\sum_{t=1}^{T} b_h e^{-rt}F(t) = \int_0^T (100 - RV_t)e^{-rt}f(t)dt \quad \text{for high liquidity traders,} \quad (12)$$

$$\sum_{t=1}^{T} b_l e^{-(r+d)t}F(t) = \int_0^T (100 - RV_t)e^{-(r+d)t}f(t)dt \quad \text{for low liquidity traders,} \quad (13)$$

where d is the additional holding cost.

From these two equations we can extract the CDS premium for each type of traders as:

$$b_h = \frac{\int_0^T (100 - RV_t) e^{-rt} f(t) dt}{\sum_{t=1}^T e^{-rt} F(t)} \quad \text{for high liquidity traders,} \tag{14}$$

$$b_l = \frac{\int_0^T (100 - RV_t) e^{-(r+d)t} f(t) dt}{\sum_{t=1}^T e^{-(r+d)t} F(t)} \quad \text{for low liquidity traders.} \tag{15}$$

Obviously, trade occurs only if $b_h < \tilde{b} < b_l$. Introducing the value of search process V, the trader has to be indifferent between searching alone or buying to a market maker. We get:

$$V = p_h b_h + p_l (V + C) = \frac{p_h b_h + C p_l}{1 - p_l}, \tag{16}$$

where C is the search cost.

The market price $\tilde{b}$ is equal to:

$$\tilde{b} = V = b_h + \frac{C p_l}{1 - p_l}, \tag{17}$$

where $\frac{Cp_l}{1-p_l}$ is the additional spread for the asset (CDS and bond that we note respectively S_{CDS} and S_{bond}).

$\tilde{b}$ is the market price for the CDS and is such that $\tilde{b} = b + S_{CDS}$. $\tilde{y}$ is the market price for the bond and is equal to $y + S_{bond}$. Taking into account liquidity, equation (11) becomes:

$$\tilde{b} = \tilde{y} - r - (S_{bond} - S_{CDS}). \tag{18}$$

B Computing P&L

Generic Bond P&L

The MtM component of the P&L of an asset is the same whatever the asset, i.e. the difference between the prices at two distinct dates. In the case of a bond, it can be expressed as the variation of the yield-to-maturity (YtM hereafter) multiplied by the sensitivity of a one unit variation. Thus, we note:

$$P\&L_t^{bond} = [YtM_t - YtM_{t-1}] \times sensi_t^{bond}. \tag{19}$$

In order to define the sensitivity, we have to specify what the duration is. This latter is the weighted average maturity of cash flows, expressed as:

$$D = \sum_{t_i=1}^{N} \frac{t_i \times \frac{CF_i}{(1+YtM)^{t_i}}}{PB}, \tag{20}$$

where t_i is the time in year until the next i^{th} payment, CF_i is the i^{th} cash flow, YtM is the Yield-to-Maturity and PB is the present value of the bond.

The sensitivity, $sensi_t^{bond}$ is defined as the opposite of the modified duration, that is, in the case of periodically compounded yields, the duration over the Yield-to-Maturity:

$$sensi_t^{bond} = \frac{D}{(1 + YtM_{\%})}. \tag{21}$$

In the case where the CDS and the Bond are not expressed in the same currencies, we need to correct the P&L of the generic bond by the corresponding exchange rate. We call PB_t the present value of the bond in local currency and X_t the exchange rate at time t. We know that the dollar price's variation is expressed as:

$$[PB_t - PB_{t-1}]_{\$} = \frac{PB_t}{X_t} - \frac{PB_{t-1}}{X_{t-1}}. \tag{22}$$

Linearizing this expression, we can separate the MtM component of the bond's P&L in two parts:

$$[PB_t - PB_{t-1}]_{\$} \simeq \frac{1}{X_{t-1}}(PB_t - PB_{t-1}) - \frac{PB_{t-1}}{X_{t-1}} \left(\frac{X_t - X_{t-1}}{X_{t-1}} \right). \tag{23}$$

The first term represents the P&L of the Bond and the second, the gain (or loss) due to the variation of the exchange rate. Crossing the expression of the P&L given in equation (19), we obtain:

$$[PB_t - PB_{t-1}]_\$ \simeq \frac{1}{X_{t-1}} \left[(YtM_t - YtM_{t-1}) \times sensi_t^{bond} \right] - \frac{PB_{t-1}}{X_{t-1}} \left(\frac{X_t - X_{t-1}}{X_{t-1}} \right). \quad (24)$$

CDS P&L

To compute the P&L of the CDS, two informations need to be recalled: the trading horizon is one day and the price of a CDS strategy at the issuance is equal to zero. Consequently, the P&L of a CDS, assuming that we neglect the carry part, equals the selling price. In the case of a CDS, we are able to express the price at time t as the product of the premium's variation between t and the issuance date, and the sensitivity to a change of 1bp of the CDS premium. Summing up, the P&L of the CDS can be expressed as:

$$[PC_t - PC_{t-1}] = PC_t = [S_t - S_{t-1}] \times sensi_t^{CDS}, \quad (25)$$

where PC_t is the price of the CDS contract at time t. As we consider $t-1$ as the date when we start the contract, PC_{t-1} is null because the value of a CDS at the opening is equal to zero.

Using a continuous time Poisson model, the sensitivity to a 1bp premium variation is equal to:

$$sensi_t^{CDS} = \int_0^T e^{-(r+\lambda)\theta} d\theta = \frac{1 - e^{-(r+\lambda)T}}{r + \lambda}, \quad (26)$$

where $\lambda = \frac{S_t}{1-RR}$ with RR is the recovery rate.

C RSDC model

According to these results, our approach is in the line of Pelletier (2006) that has been used in the context of portfolio allocation [see Giamouridis and Vrontos (2007)]. It allows in particular to decrease the number of variance parameters to consider. Our model is a combination of a mixture model for the correlation matrix and a Threshold GARCH model [or TGARCH, Zakoian (1994)] to take into account asymmetric volatility dynamics. However, our estimation method imposes to assume that the heteroscedasticity is asset specific and not common across assets.

Note the K asset returns are defined by:

$$r_t = H_t^{1/2} U_t, \tag{27}$$

where $U_t |\, \Phi_{t-1} \sim iid(0, I_K)$, U_t is the $T \times K$ innovation vector, and Φ_t is the information available up to time t.

The conditional covariance matrix H_t is decomposed into [Bollerslev (1990) or Engle (2002)]:

$$H_t \equiv S_t \Gamma_t S_t, \tag{28}$$

where S_t is a diagonal matrix composed of the standard deviation $\sigma_{k,t},\ k = 1, \cdots, K$ and Γ_t is the $(K \times K)$ correlation matrix. Both matrices are time varying.

The conditional variance may follow a TGARCH(1,1) such that:

$$\sigma_{i,t} = \omega_i + \alpha_i^- min(\epsilon_{i,t-1}, 0) + \alpha_i^+ max(\epsilon_{i,t-1}, 0) + \beta_i \sigma_{i,t-1}, \tag{29}$$

where ω_i, α_i^-, α_i^+ and β_i are real numbers.

Under assumptions of: $\omega_i > 0$, $\alpha_i^- \geq 0$, $\alpha_i^+ \geq 0$ and $\beta_i \geq 0$, $\sigma_{i,t}$ is positive and could be interpreted as the conditional standard deviation of $r_{i,t}$. However, it is not necessary to impose the positivity of the parameters and the conditional standard deviation is the absolute value of $\sigma_{i,t}$.

The correlation matrix is defined as:

$$\Gamma_t = \sum_{n=1}^{N} \mathbb{1}_{(\Delta_t = n)} \Gamma_n, \tag{30}$$

where $\mathbb{1}$ is the indicator function, Δ_t is an unobserved Markov chain process independent from U_t which can take N possible values ($\Delta_t = 1 \cdots, N$) and Γ_n are correlation matrices. Regime switches are assumed to be governed by a transition probability matrix $\Pi = (\pi_{i,j})$, where

$$Pr\left(\Delta_t = j \middle| \, \Delta_{t-1} = i\right) = \pi_{i,j}, \; i, j = 1, \cdots, N.$$

This approach allows to discriminate between on the one hand the volatility dynamics through S_t and on the other one the correlation dynamics through the state variable Δ_t.

D Estimation of RSDC

The estimation of this model is made using a two-step procedure: (i) the univariate estimation of standardized residuals with GARCH or TGARCH models and maximum likelihood and, (ii) the estimation of correlation matrices and probabilities to be in state n ($n = 1, \cdots, N$) with an EM algorithm (Dempster et al. (1977)). Using this method is more tractable when the number of observed series is more than a few. Indeed, the number of parameters could become very large and the one-step likelihood maximisation becomes intractable.

We should introduce θ, the complete parameter space that we split in two parts with: θ_1 that corresponds to the parameter space of the univariate volatility model and θ_2 that corresponds to the parameter space of the correlation model. We compute the log-likelihood taking a correlation matrix equal to the identity matrix. In other words, we estimate univariate TGARCH model for each asset.

D.1 First Step

To model the full covariance matrix, we estimate the standard deviations and the correlations separately. This first step focus on the estimation of standard deviations.

The parameters of univariate TGARCH model are estimated with maximum likelihood, taking the case of a TGARCH(1,1), as presented in section 1. We have to specify the distribution of U_t in order to estimate the likelihood function that we want to maximize. In our case, U_t are *iid* and normally distributed [$U_t \sim \mathcal{N}(0, 1)$] allowing to consider Gaussian likelihood. However, we don't make the assumption that is the true law of U_t.

Note $\theta_1 = (\omega, \alpha^-, \alpha^+, \beta)$. As a result, the Gaussian likelihood is:

$$L(\theta_1) = L(\theta_1; r_0, \cdots, r_T) = \prod_{t=1}^{T} \frac{1}{\sqrt{2\pi\tilde{\sigma}_t^2}} exp\left(-\frac{r_t^2}{2\tilde{\sigma}_t^2}\right) \tag{31}$$

with $\tilde{\sigma}_t$ are obtained recursively ($\forall t \geq 1$) as:

$$\tilde{\sigma}_{i,t} = \omega_i + \alpha_i^- min(\epsilon_{i,t-1}, 0) + \alpha_i^+ max(\epsilon_{i,t-1}, 0) + \beta_i \tilde{\sigma}_{i,t-1}$$

Taking the logarithm, we have to minimize the log-likelihood $\tilde{l}_t(\theta_1)$ defined as:

$$\tilde{l}_t = \tilde{l}_t(\theta_1) = \frac{r_t^2}{\tilde{\sigma}_t^2} + log(\tilde{\sigma}_t^2)$$

Thus, $\hat{\theta_1}$ is the solution of:

$$\hat{\theta_1} = \arg\min_{\theta_1} \frac{1}{T} \sum_{t=1}^{T} \tilde{l}_t(\theta_1) \tag{32}$$

After the estimation of parameters, we get the standardized residuals, noted $\tilde{U}_t$ as:

$$\tilde{U}_{i,t} = \frac{r_{i,t}}{\tilde{\sigma}_{i,t}}$$

In the next step, we use it to estimate the correlation matrices. We introduce a regime switching adding dynamic correlations. It measures the probability to be in the state n (in our case $n = 0, 1$ corresponding respectively to liquid and illiquid states).

D.2 Second Step

In this second part of estimation of our model, we use the Expectation Maximization algorithm (EM thereafter). The main advantage is the ability to take into account high number of parameters coming from each Γ_n.

EM Algorithm

This algorithm is presented in Hamilton (1994, chapter 22). We have to estimate the vector of parameters θ_2:

$$\tilde{\theta}_2 = \arg\min_{\theta_2} \left[\frac{1}{2} \sum_{t=1}^{T} K log(2\pi) + log(|\Gamma_t|) + \tilde{U}_t' \Gamma_T^{-1} \tilde{U}_t \right] \tag{33}$$

The number of parameters increases at a quadratic rate with the number of asset returns. As a consequence, to realize these estimation, we use EM algorithm that has no restrictions on the number of parameters.

Then, Hamilton (1994, chapter 22) expose that Maximum Likelihood estimates of the transition probabilities (i) and the correlation matrices (ii):

$$(i) \quad \tilde{\pi}_{i,j} = \frac{\sum_{t=2}^{T} P\left[\Delta_t = j, \Delta_{t-1} = i | \underline{\tilde{U}_T}; \tilde{\theta}_2\right]}{\sum_{t=2}^{T} P\left[\Delta_{t-1} = i | \underline{\tilde{U}_T}; \tilde{\theta}_2\right]} \tag{34}$$

$$(ii) \quad \tilde{\Gamma}_n = \frac{\sum_{t=1}^{T} (\tilde{U}_t \tilde{U}_t') P\left[\Delta_t = n | \underline{\tilde{U}_T}; \tilde{\theta}_2\right]}{\sum_{t=1}^{T} P\left[\Delta_{t-1} = n | \underline{\tilde{U}_T}; \tilde{\theta}_2\right]} \quad \text{for } n = 1, 2 \tag{35}$$

Estimates of transition probabilities are based on the smoothed probabilities. We could see that $\tilde{\Gamma}_t$ is not directly a correlation matrix. It must be rescaled because their diagonal elements are not constrained to be equal to one. Off-diagonal elements are between -1 and 1. This step is needed because the product of standardized residuals is not constrained to have elements between -1 and 1. Then we rescale Γ_t at each iteration as:

$$\Gamma_t = D_t^{-1} \tilde{\Gamma}_t D_t^{-1} \tag{36}$$

where D_t is a diagonal matrix with $\sqrt{\tilde{\Gamma}_{i,i,t}}$ on row n and column n.

The algorithm starts with initial values $\tilde{\theta}_2^{(0)}$ for the vector θ_2. With $\tilde{\theta}_2^{(0)}$ we can compute a new vector $\tilde{\theta}_2^{(1)}$ based on equations (34) and (35). The algorithm works until the difference between $\tilde{\theta}_2^{(m)}$ and $\tilde{\theta}_2^{(m+1)}$ is less than a defined threshold.

Computation

We develop in this subsection the method to compute the EM algorithm. The elements of the transition probabilities matrix, $\tilde{\pi}_{i,j}$ are defined as the ratio of consecutive probabilities ($P[\Delta_t = j, \Delta_{t-1} = i|\tilde{U}_t, \theta_2]$) and the probabilities to be in state j at time t. They are obtained iteratively from $t = 1$ to T.

Note that, conditional probability is defined by [see Hamilton, (22.3.7)]:

$$P[\Delta_t = j|\tilde{U}_t, \theta_2] = \frac{\pi_j \times f(\tilde{U}_t|\Delta_t = j, \theta_2)}{f(\tilde{U}_t|\theta_2)} \tag{37}$$

where $f(\tilde{U}|\Delta_t = j, \theta_2)$ is the probability density of the multivariate normal distribution with zero mean and Γ_j as covariance matrix, evaluated for $\tilde{U}_t$.

With equation (37), we compute probabilities at time $t = 1$. Then, we compute consecutive probabilities recursively:

$$P\left[\Delta_t = j, \Delta_{t-1} = i|\tilde{U}, \theta_2\right] = P\left[\Delta_{t-1} = i|\tilde{U}, \theta_2\right] \times P\left[\Delta_t = j|\tilde{U}, \theta_2\right] \times \pi_{i,j} \tag{38}$$

where $P\left[\Delta_t = j|\tilde{U}, \theta_2\right] = f(\tilde{U}|\Delta_t = j, \theta_2)$.

Then, conditional probabilities to be in state j at time t are obtained making the ratio of the sum of the two consecutive probabilities of being in state j at time t and the sum of all consecutive probabilities.

Introduce the notation $\xi_{t|\tau}$, the $(N \times 1)$ vector whose j^{th} element is $P[\Delta_t = j|\tilde{U}_\tau, \theta_2]$. This notation allows to present two cases of $\xi_{t|\tau}$: (i) for $t > \tau$ it represents a forecast about the regime and (ii), for $t < \tau$ it represents the smoothed inference (about the regime in date t based on data obtained through some later date τ). We focus on smoothed probabilities that is defined by:

$$\tilde{\xi}_{t|\tau} = \tilde{\xi}_{t|t} \odot \{\Pi' \cdot [\tilde{\xi}_{t+1|T}(\div)\tilde{\xi}_{t+1|t}]\} \tag{39}$$

Smoothed probabilities are obtained iterating on backward for $t = T, T-1, T-2, \cdots, 1$. We come back from equation (38) to compute consecutive probabilities with smoothed probabilities. Then, we compute $\theta_2^{(m)}$ with equation (34) and (35) rescaling at each iteration the correlation matrix with equation (36).

The breaking rule of the algorithm is defined by the fact that the correlation matrix computed by the last iteration is almost equal to the previous correlation matrix. We have to define a threshold under which, we consider that matrices are equal.

Initialisation of the Algorithm

To start the algorithm, we have to choose the space of initial parameters, $\theta_2^{(0)}$. In this space, we input correlation matrices for each state of our model (in our case, two). The algorithm starts with one correlation matrix of the state (1) equal to identity matrix. For the second state, we use the Gramian matrix method (Holmes (1991)) to generate random correlation matrix. Note that a correlation matrix has to be defined semi-positive with diagonal elements that are equal to one and off-diagonal elements that are between -1 and 1. We use the Gramian matrix $T'T$ where $T := (t_1, \cdots, t_K)$ and t_i is the i^{th} column. Then, we normalize the matrix as: $t_i = \tau_i/||\tau_i||$.

For a K-variate process, we generate K independent pseudo-random vectors normally distributed, τ_i.

E Correlations Matrices

	Brazil	Chile	Hungary	Mexico	Poland	Russia	South Africa	Thailand	Turkey
Brazil		0.452	0.364	0.877	0.313	0.608	0.522	0.304	0.673
Chile	0.608		0.312	0.490	0.287	0.381	0.345	0.310	0.350
Hungary	0.535	0.440		0.410	0.588	0.537	0.570	0.317	0.503
Mexico	0.940	0.634	0.551		0.338	0.621	0.534	0.318	0.662
Poland	0.579	0.464	0.773	0.570		0.456	0.489	0.357	0.427
Russia	0.634	0.520	0.759	0.637	0.743		0.747	0.436	0.795
South Africa	0.619	0.532	0.746	0.614	0.744	0.883		0.390	0.716
Thailand	0.400	0.332	0.425	0.388	0.412	0.479	0.459		0.402
Turkey	0.675	0.534	0.745	0.654	0.742	0.904	0.890	0.479	

Table 9 – Correlations matrices of the two regimes for the CDS market. The upper part of the matrix corresponds to the regime 1 while the lower part corresponds to the regime 2.

	Brazil	Chile	Hungary	Mexico	Poland	Russia	South Africa	Thailand	Turkey
Brazil		0.270	0.396	0.482	0.293	0.188	0.432	0.010	0.378
Chile	0.329		0.285	0.244	0.335	0.216	0.313	0.062	0.296
Hungary	0.560	0.396		0.331	0.657	0.528	0.509	0.005	0.472
Mexico	0.582	0.404	0.586		0.217	0.096	0.435	0.044	0.326
Poland	0.603	0.412	0.869	0.646		0.614	0.348	0.092	0.407
Russia	0.385	0.381	0.558	0.412	0.542		0.219	0.078	0.269
South Africa	0.596	0.393	0.688	0.636	0.718	0.497		0.036	0.513
Thailand	0.224	0.209	0.277	0.250	0.285	0.268	0.280		0.046
Turkey	0.591	0.413	0.700	0.612	0.712	0.499	0.694	0.312	

Table 10 – Correlations matrices of the two regimes for the sovereign debt market. The upper part of the matrix corresponds to the regime 1 and the lower part corresponds to the regime 2.

F Robustness Check

		GARCH(1,1)	TGARCH(1,1)	TGARCH(2,2)
	Brazil	13.29		
		0.35		
	Chile	3.14		
		0.99		
	Hungary	**34.41**	**29.19**	18.89
		0.00	**0.00**	*0.09*
	Mexico	11.44		
		0.49		
CDS	Poland	6.62		
		0.88		
	Russia	9.18		
		0.69		
	South Africa	13.87		
		0.31		
	Thailand	15.93		
		0.19		
	Turkey	3.27		
		0.99		
	Brazil	12.40		
		0.41		
	Chile	10.16		
		0.60		
	Hungary	12.49		
		0.41		
	Mexico	18.48		
		0.10		
Bond	Poland	8.47		
		0.75		
	Russia	15.55		
		0.21		
	South Africa	13.19		
		0.36		
	Thailand	18.51		
		0.10		
	Turkey	9.01		
		0.70		
	Brazil	14.31		
		0.28		
	Chile	3.25		
		0.99		
	Hungary	**35.09**	**30.26**	19.58
		0.00	**0.00**	*0.08*
	Mexico	3.31		
		0.99		
Basis	Poland	13.95		
		0.30		
	Russia	9.47		
		0.66		
	South Africa	16.82		
		0.16		
	Thailand	12.32		
		0.42		
	Turkey	9.54		
		0.66		

Table 11 – ARCH effect test on standardized residuals from GARCH(1,1) model and alternative models in case of heteroscedasticity. The test is realized for a risk threshold of 5% and a number of lags equal to 12. In other words, we test whether all ρ are equal to 0 in the following equation: $u_t^2 = \alpha + \sum_{p=1}^{P} \rho^p u_{t-p}^2 + \epsilon_t$, $\forall p = 1, \cdots, 12$.